Definiteness: Two Solutions

James Stevens
University of Minnesota

1 Introduction

Definite expressions include those linguistic forms that signal something in particular. Examples include "Marco Polo," "the Armenians," and "our society". In each of the above, one can isolate which things correspond with the expression, to the exclusion of everything else. **Indefinite expressions** do not signal anything *in particular* but signal something *categorically*. If one says "some Armenians" or "a spade," one is not typically expected to infer any specific Armenians or spade.

If we consider that we need to encode aspects of the world into our brains in order to behave with some success, then any model of the world must allow for a means to handle expressions referring *in particular* and those expressions referring *categorically.* In what follows, I will treat different approaches to **reference**, that is, what is being signaled. A denotational model treats truth value as primitive, which holds for entities with stated properties. Another approach to consider holds a referent to be an indicated collection of features that yield correlated responses. I will then propose that reference is inferred probabilistically by means of **attention** to magnify the probability of the most likely referents.

Philosophy has responded to the problem of definiteness, among which Bertrand Russell's formal analysis of the *uniqueness* of a definite expression (Russell 1905). Various criticisms of Russell's analysis ensued. Strawson delivers criticism of the effectiveness of Russell's analysis to capture language use (Strawson 1950). Peacocke discusses how language can be used irrespectively of its truth (Peacocke 1975). Donnellan indicates the problem of **incompleteness** in Russell's semantics (Donnellan 1966). I will address these criticisms with a solution that employs an implicit set to restrict the domain of inference.

Problems with my offered solution will turn our discussion to psychological work on attention as a means to an improved analysis. Attention represents an important body of research in psychology, and yet definitions of attention are plentiful enough to make it unclear as an object of study. According to one assessment, attention involves differential neural activity between different stimuli, that is, one concentrates one's thought on some things more than others. Such considerations will highlight how to represent the value of linguistic signs.

By considering language use to depend on probabilistic reasoning, the effect one's attention can be calculated as an increase in probability of a certain referent. This allows for the inference of a referent, given incomplete description. The latter solution offers a more economical and robust approach to reference.

2 Philosophical treatment of definiteness

A principal difficulty with arguing for a proper analysis of linguistic phenomena, such as of definiteness, is that different analyses seek to explain different aspects of a phenomenon and naturally for different purposes. Bertrand Russell wrote a highly influential paper 'On Denoting', which set up a framework by which language can be viewed with the same logical mechanics that are essential to mathematics (Russell 1905). I will discuss this paper and its bearing on definiteness shortly but firstly will look at its assumptions.

Russell based his work partly on his contemporary Gottlob Frege's work 'On Sense and Nominatum', in which Frege divided the ways in which things can be the same as those sharing *sense* or *nominatum* (Frege 1949). Those things that share their *sense* are equal analytically, that is, no knowledge of the world is necessary to draw a conclusion of sameness, because the related things are the same *a priori* by virtue of their form, i.e. $a=a$. Things can also be the same but are not recognized as such by the form. We can conclude from knowledge of the world if two forms refer to the same thing, i.e. that they share a *nominatum*, $a=b$.

Russell took the division of *denotation* and *meaning* (as he respectively called nominatum and sense) as useful in the logical treatment of natural language. Because we do not by nature know everything, we need to learn of some things by report. When we inform each other of the state of the world, we use *denoting phrases*, which are forms that indicate something of the world. Denoting phrases indicate some referent by means of a linguistic expression. According to Russell,

> [D]enoting phrases never have any meaning in themselves, but [every] proposition in whose verbal expression they occur has a meaning. (Russell 1905)

In the above quote, Russell makes clear that the meaning we gain from an expression is the propositional content, an explicit connection of language to logic that he shared with Frege.

2.1 Russell: definiteness as uniqueness

By his theory of denotation, Russell sought to describe cases in which expressions are used to designate some object. Names are the clearest example of this occurrence, as 'Elizabeth II' for instance does indicate one individual. Other definite descriptions often do much the same, as when they can be used only to point out something unique, as in the phrase 'the author of *Waverly*'. Definite descriptions may well be used as expressions to help indicate a unique object, e.g. 'the horse', 'she', but these expressions themselves cannot be expected to do this alone. These latter uses of definite descriptions are not treated by Russell, as they involve more than a correspondence between linguistic form and reference.

Russell argued that definite descriptions, when used strictly (as explained above), are used as a complex meaning to denote a single object. Any expression of the type *the* ϕ means:

(1) There exists at least one individual *x* that makes ϕ true.
For any individual that makes ϕ true, that individual is identical to *x*.

For example, the expression 'the author of *Disgrace*' is a definite description, of the type the ϕ, and we see its use as a denoting phrase in the following sentence:

(2) The author of *Disgrace* is JM Coetzee.

We can demonstrate the truth of the proposition expressed by the sentence above by Russell's expansion of definite phrases into complex names:

(3) There exists at least one individual *x* who is 'the author of *Disgrace*'.
For any individual who makes 'the author of *Disgrace*' true, that individual is identical to *x*, where *x* = Coetzee.

Out of the above conditions we see that definite descriptions will indicate some unique entity, as there is only one individual who is the author of *Disgrace*. A formal expression of this shows the added advantage that this resembles Frege's logical formulation for the number *one* (Frege 1884):

(4) $\exists x$ Author-of-*Disgrace* x
$\forall y$ Author-of-*Disgrace* $y \rightarrow y = x$

There has been much criticism of Russell's ideas above, but it should be borne in mind that Russell is describing that aspect of language which designates and is therefore pliable to logical analysis. That Russell's formulation does not capture all definite usage cannot be a well-placed fault.

2.2 Strawson: where is the meaning?

Peter Strawson famously challenged Russell's claims in his article 'On Referring', in which Strawson questions how much denotation figures into language as it is used (Strawson 1950). Strawson argued against Russell's treatment of certain sentences with definite descriptions, e.g. 'The present king of France is wise', as having a truth value. To Strawson, it is an error to take the sentence to be true or false without regard to how the sentence is used. Thus, he separates language *cum* expression and language in its use:

(5) a. a sentence
b. a use of a sentence
c. an utterance of a sentence

By the above distinctions, a sentence such as 'The present king of France is wise' alone does not relate to truth, as it must be uttered by an individual to be considered a faithful account of that individual's world or not. If someone alive during the reign of Louis XIV were to utter the sentence, a hearer would be free to judge the truth of the expressed proposition. Someone alive today, however, could not use the same sentence to refer to Louis XIV or to any individual, as France has no king, and thus we cannot assign truth to a proposition if expressed in a sentence for which the use is undetermined.

According to Strawson, therefore, language can be rightly interpreted only in the context of being used by someone. Reference does not happen on its own in a sentence but is something that someone uttering the sentence does. And, again, sentences are not true or false, but rather assertions of a sentence are true or not. Although Strawson's observations sound faultless when differentiating language from its use, it is not clear to me how this speaks against Russell. Russell's paper, after all, did not purport to describe the general use of speech, which he considered too vague for logical analysis (Russell 1957). Russell's program was to analyze the propositions found in sentences, and these propositions themselves express a truth value. It is an altogether different task to determine the *meaning* of

these sentences, which is a product of linguistic expressions and other cues (i.e. context). Unfortunately, both Strawson and Russell use the word ‘meaning’, among other words bound to get entangled in confusion. The lesson that I have taken from Strawson is that we interpret language by its function within a context. Whether this contradicts Russell is not clear and perhaps unimportant at this point. Since my stated interest is how we understand each other’s use of language, and definite phrases in particular, I will not seek the same objectives as Russell and Frege and will have to find an alternative to denotation. Until then, I will consider other criticisms of Russell’s theory.

2.3 Incompleteness: just not unique enough

Strawson, among others (e.g. Peacocke 1975), pointed out another concern for how definite descriptions are treated by Russell. Quite often, a definite description does not uniquely indicate a referent, although the use of such descriptions does serve to pick out a unique referent. This apparent paradox comes from the incompleteness of some definite descriptions. Consider the expression "the soup", as in the sentence "The soup is too watery." This sentence could be used to make a statement about a certain spatio-temporal set (such as what is inside a pot on the stove). The cue to tell use what in the world is too watery is the word 'soup', but this sentence most certainly does not opine about all soup. By some means, the hearer understands that only some things and not other things are intended by a definite description, although the linguistic expression itself does not complete this understanding. The expression 'the author of *Disgrace*' does uniquely signal a referent, that is, the South African writer JM Coetzee. I might refer to Coetzee by various other linguistic means: 'the South African writer', 'the Australian resident', 'him', or 'Coetzee' that do not fully specify the referent.

This criticism speaks to the strange consequence that sentences such as 'The soup is too watery' would be false by Russell's analysis. There would not be a single entity that meets the conditions for definiteness:

(6) There exists one individual *x* that makes 'soup' true.
For any individual that makes 'soup' true, that individual is identical to *x*.

Most uses of the expression 'the soup' would not imply that there exists nothing else that is soup than what is being signaled. To Russell, this is another problem, that of egocentricity. The fact that people talk about things that are unique within their experience is orthogonal to the truth value of a proposition expressed in their language. If we grant Russell the separability of language as symbolic expression and its use within human experience, we will need some means to account for the

latter satisfactorily such that our sentence 'The soup is too watery' is true again. Otherwise, the conditions that are offered here will not be very useful, if egocentricity is permitted to supersede the conditions randomly.

2.4 Donnellan and the two faces of definiteness

Keith Donnellan took issue with both Russell and Strawson for not appreciating the two functions of definite descriptions, which he calls the *attributive* and *referential* uses (Donnellan 1966). The attributive use includes statements about any referent that fits the description. Donnellan's illustrative examples include one hearing that a very lovable person, Smith, has been murdered. The thought of anyone murdering Smith is so unfathomable that one exclaims, 'Smith's murderer is insane.' In this instance, we have no idea who Smith's murderer is in the flesh, as it could well have been a group of deranged individuals. The other use of a definite description, the referential use, serves to pick out an individual by a linguistic expression. The stated linguistic expression works simply as a device to cue our audience to whomever we have in mind and well could be replaced by different words. In the trial of Smith's murder, in Donnellan's example, one might be seated in the courtroom and look over to the defense table. There, slobbering and pulling at his hair is an insane-looking individual. We might nudge our neighbor and tell her such, assuming that the crazed person to be the defendant. In this case, we state, 'Smith's murderer is insane' to mean something equivalent to 'That person over there who must be Smith's murderer just by how guilty he looks is insane'. Interestingly, this second use of the same sentence might have been used to describe not Smith's murderer, but simply whom the speaker *thinks* to be Smith's murderer and who might in fact be the defense attorney.

Donnellan's distinction between attributive and referential uses of definite descriptions brings up the problem of what it means for a sentence to express a proposition. The attributive use shows little problem in the conveyance of a proposition, as it is within this sense that Russell wrote. However, do we consider interpret speech simply by the words that are expressed or by what the speaker intends to communicate? According to the latter, a statement could be understood, even though the indicated referent does not in fact make the proposition true. It did not seem to be Donnellan's claim that speaker intention is the sole basis of language interpretation, as the attributive use surely does play a part. What he takes issues with is the limited applicability of an analysis that does not differentiate the uses of similar linguistic expressions.

Although Donnellan's observations do bring up a problem with the relationship between form and reference, the claimed two uses of definiteness are not independent. Namely, the referential use cannot work except by way of

attributive use. The example of referring to the lawyer as 'Smith's murderer' might actually communicate a message successfully, but it would do so by mistaken understanding. The words in the expression still predominately work, by virtue of their meaning, to signal the correct referent. The referential use cannot succeed without the availability of the attributive use. In the case of mistaking the defense attorney for Smith's lawyer, it is only other contextual cues that influence a conclusion that the man could be a murderer. Another example of the referential use is when we want to pick out an interesting-looking man at a party by the expression 'the man drinking a martini'. If, as it turns out, there was only water in the man's martini glass, the expression still succeeds at referring to him, but the success of this false attribution happens only because of the high statistical association of the words with the visual cues of a man and a martini glass.

2.5 A synthesis of arguments

We might consider the attributive and referential uses of definiteness to reflect two types of relationship between the referent and its identifying property, which I will call an **open-set** relationship and a **closed-set** relationship, by which I differentiate those relationships that are open or closed to an exchange of referent.

The attributive use of definiteness marks an open-set relationship, in that the identifying property signals any referent that satisfies the expressed property. If, for instance, I say something about the King of Spain, such as that he has dominion over vast land estates in his country, this proposition might be true today of only individual, Juan Carlos. Upon some future heir's succession to the throne, however, the same proposition might be true of a different king at this different time. The distinguishing aspect is that the set of individuals who are king of Spain is kept maximally at one, but whoever materially belongs to this set is not who is intended by an open-set relationship: The statement refers to any one person x who embodies the open set King-of-Spain(x). This is similarly the case in Donnellan's examples of 'Smith's murderer' or 'the man drinking a martini', where the situations permit statements about anyone who would kill Smith or anyone drinking a martini, despite pretenses to sobriety. The speakers care nothing for which individuals fill the role of Smith's murderer or martini-drinking, only that conclusions can be drawn from any individual who does such.

The referential use of definiteness is conversely a closed-set relationship, in that there is no possible exchange of the indicated individual. If the person being indicated is Juan Carlos, the present Spanish monarch, then there exists only one individual, which is upon itself a closed singleton set $\{a\}$, that we are discussing. a can be signaled by such attributes as Juan-Carlos(a) and King-of-Spain(a) that are possibly sufficient to inform us of the referent a.

What advantage does it confer me to rephrase attributive and referential uses as open and closed set relationships? Firstly, it at least alludes to an explanatory difference, in that these two uses can be treated generally according to whether the speaker is in fact making reference to the expressed feature set (i.e. anything that satisfies the properties: attributive use) or is referring to a particular individual by way of some feature set, i.e. referential use. The metaphor of open or closed illustrates a mechanism of the two relationships, namely that a closed-set relationship is conditioned on the referent having previously been in an open-set one, as was mentioned above (2.4) with Donnellan's examples, that the referential use of definiteness depends on the availability of the attributive use. Relationships between a property and referenced elements must be established originally as open to allow for the categorization of these elements, later potentially closed if a set is defined on those members.

What generalizations are available from the distinction of open- and closed-set relationships? Importantly, closed-set relationships allow for incomplete description, whereas open-set ones do not. Closed-set relationships, because they are closed around a particular number of individuals, allow for the referenced individuals to be indicated by various means, none of which are necessarily uniquely associated with those individuals. In the more likely employment of an incomplete description, some context or, as Russell calls it, *egocentricity* is required to establish what is being referenced. Open-set relationships, because the expressed properties denote whatever referent fits the description, cannot indicate additional individuals than the definite form (marked for number) allows. Perhaps I assert the following:

(7) The King of Spain lives in Madrid.

If in an open-set relationship (i.e. attributive use), the above statement would be a general statement about whoever is king. It cannot be an incomplete description, because this would imply that more than one individual makes the statement true, but the definite singular form does not correspond to this plural sense. If in a closed-set relationship (i.e. referential use), then incomplete descriptions are possible, because the expressed properties *King of Spain* are simply means to signal reference to a particular individual, which could be indicated differently:

(8) *Juan Carlos* lives in Madrid.
He lives in Madrid.
That man who sits on horses lives in Madrid.

The consequences for Russell's treatment of definite descriptions, I believe, are the following. Russell formulized definite descriptions by their

uniqueness, provided that the *strict* reading of definiteness is considered: that only one such individual exists. However, if we accept this provision, then the uniqueness property is covered redundantly by Russell's provision and the following logical form:

(9) $\forall y$ King-of-Spain $y \rightarrow y = x$

If we do not follow his provision and adhere only to the logical form, then the proposition will be false for any entity expressed by a property that is not unique to it. Incomplete descriptions did not interest Russell as a counter-example, because he readily conceded the role of egocentricity to restrict the referent. Thus, 'the guy' could still be uniquely referential if one's attention is focused on only one individual. Since egocentricity is necessary for incomplete descriptions, which occur in closed-set relationships, the stipulations are (1) is neither necessary nor sufficient for the inference of a referent, it being the task of egocentricity to restrict the possible referents. Since this process is considered necessary for definiteness to be resolved, there seems less reason to logically encode a uniqueness provision that alone is either possibly untrue (in referential use) or redundant (in attributive use, where the form alone picks out a unique referent). Egocentricity alone can contribute the uniqueness provision.

An additional consideration is that of number. According to Russell's formulation of definiteness, only a single individual can be denoted by a definite expression. Plural definite descriptions abound (e.g. 'the neighbors', 'my poodles'), and this should be treated somehow. The restriction of uniqueness could be applied over sets rather than entities.

2.6 A solution

Above it was argued that it is not necessary to incorporate uniqueness into the logical form, because egocentricity is already necessary for a unique referent to be inferred. This treats cases of incomplete description and renders the uniqueness provision redundant. We might posit a logical form anew that incorporates egocentricity or, as I prefer, **attention**. I use attention to imply the differential amounts of processing stimuli that do not correlate to original differences in the stimulus quantities. For example, there might be two cats in my kitchen. Since I am accustomed to there being at most one cat in my kitchen (my cat Peepers), a second cat would be peculiar. In this situation, I would undoubtedly attend to the second cat more than Peepers, even if the stimuli related to seeing the strange cat were no stronger than those from seeing Peepers.

If it is accepted that our understanding of language takes into account one's attention, how to incorporate this might not be yet straightforward. One solution could be to take one's attention on a referent as itself a distinguishing property of that referent. By this, we can formulate definite descriptions of the type *the* ϕ analogously to those of the type *all* ϕ. Here below, *p* is a property assigned by the speaker's attention:

(10) $(\forall x)[\text{bird}(x) \wedge \text{p}(x) \rightarrow \text{red}(x)]$
The bird(s) is/are red.

This logical form does not specify for number but can be so stipulated:

(11) $(\forall x)[\text{bird}(x) \wedge \text{p}(x) \rightarrow \text{red}(x)]$
$\wedge (\exists y)(\forall \text{z})[\text{bird}(y) \wedge \text{p}(y) \wedge \text{bird}(z) \wedge \text{p}(z) \rightarrow z = y]$
The bird is red.

(12) $(\forall x)[\text{bird}(x) \wedge \text{p}(x) \rightarrow \text{red}(x)] \wedge (\exists y,z)[\text{bird}(y) \wedge \text{p}(y) \wedge \text{bird}(z) \wedge \text{p}(z) \wedge z \neq y]$
The birds are red.

2.6.1 Arguments for Solution 2.6

It is advantageous to treat definiteness as not bound to number, as in Russell's theory, which could not treat definite plurals, because definiteness was considered to describe a single entity, rather than a single set.

Importantly, the present proposal handles definiteness as a contextual restriction of sets. By this method, reference is understood from signs with relatively constant value (e.g. 'bird') and others reassigned value per usage (e.g. 'the'). The pragmatic principles of inference are represented in the logical form but without corresponding value. In this way, semantic and pragmatic principles can be considered and investigated separately. The crucial difference is in the availability of a value for a sign, as pragmatically-resolved signs are represented simply as place-holders of a set, to be inferred by one's attention in the context.

Another advantage to this analysis is that it handles the complementary distribution of pronouns and articles, as illustrated in Table 1. A pattern across languages is that phrasal pronouns (e.g. 'I', 'it', but not 'my') and articles do not co-occur within the most minimal noun phrase, and this pattern could be explained in part by considering them as allomorphs. Moreover, the form of definiteness determines whether a noun is expressed.

Table 1: Conditional probability of a noun based on the definiteness form

	Noun present	Noun absent
Articles	100% (e.g. 'the bird' as NP)	0% (e.g. 'the' as NP)
Phrasal Pronouns	0% (e.g. *'it bird' as NP)	100% (e.g. 'it' as NP)

This variation between phrasal pronouns and articles as a class would be captured in the proposed model for definiteness, in that pronouns and articles would show the same semantic value, namely, variable. Of course, some conditions would need to be added to control that pronouns and articles can determine different grammatical features. English, for example, marks gender on pronouns (e.g. 'she', 'it') but not on the definite article. If this be ignored for present purposes, 'it' and 'the' differ only in whether additional propositional content is expressed:

(13) $(\forall x)[\text{parasol}(x) \wedge \text{p}(x) \rightarrow \text{striped}(x)]$
The parasol is striped.

(14) $(\forall x)[\text{p}(x) \rightarrow \text{striped}(x)]$
It is striped.

2.6.2 Arguments against Solution 2.6

This treatment of definiteness as a restriction set creates a huge inference problem. Any interpretation would need to consider an infinite number of sets that could possibly comprise the contextually-assigned predicate before choosing the intended message. One might argue against this criticism by saying that, yes, the inference involved is substantial, but this is indeed what human language learners need to do to understand a specific referent. To this counter-argument, I would ask how one's attention in a context could form a deterministic predicate; this task seems more easily handled as a probabilistic behavior. There is something awry in saying that we do not know the value of a referent without a context, and that the solution lies in one's attention in a context having propositional value. Where does this value come from? It looks likely that the employment of a restriction set is serving as an *ad hoc* measure, the nature of which remains mysterious. How does the fluid and gradient state of one's attention act as an expressible, discrete property? Although this is all in theory possible, the mechanism behind the conceptualization of one's attention would need to be explored.

An additional concern imbibing this entire discussion is the appropriateness of denotation as a device for considering human language usage. The utility of denotation to establish truth notwithstanding, it is not clear that

human language interpretation involves an assessment of sentences on the basis of truth conditions. If, however, we replace denotation with another value system for language, immediate problems must be addressed of meaning, reference, etc.

3 Toward a treatment of language interpretation

Attention is not a straight-forward phenomenon to describe, and there will understandably be dispute as to the best approach. Research from visual behavior has offered as a description that attention involves the increased processing of a visual stimulus when the stimulus itself has not intensified in luminosity (Moore & Armstrong 2003). This depiction can be extended easily to linguistic behavior in a way that avoids the awkward use of an attention property. By observing linguistic behavior, we can also consider attention in a model to exhibit an increase in response to a stimulus source that does not output a correlated increase in stimulus.

How to model this? One possible model for language usage considers a relation between form and some associated bundle of features as probabilistic. In this way, there is some probability that the word 'chocolate' refers to certain features, these forming the source of sensory cues. In order to incorporate definite expressions into such a model, we can consider the probabilities of referents held in attention to carry a higher probability than those that are not in attention.

A Bayesian model of inference is proposed here as a formal system to treat the probabilities underpinning language usage. A Bayesian model benefits from advances in Artificial Intelligence that makes explicit use of such (Stuart & Russell 2003), as well as the hypothesis within neuroscience that human cognition behaves similarly (Knill & Pouget 2004). The model depends on Bayes' Law, by which one can infer a conditional probability by the following:

(15) $$P(X \mid Y) = \frac{P(Y \mid X) \cdot P(X)}{P(Y)}$$ (Bayes 1763)

Thanks to the above equality, we can make conclusions as to the value of an unknown variable that is generating a known variable. For example, if we hear the word 'Juan Carlos,' we would likely want to know which individual X is being referred to by the sign Y. Let us allow the following known information. We might know that of all individuals known, we have heard the individual Juan Carlos being referred to as 'Juan Carlos' 20% of the time. The next most probable referent, perhaps my neighbor Nick, is referred to as 'Juan Carlos' with a probability .0002%, a hundred thousand times less likely. The prior probability that I might talk about my neighbor, $P(X_{Nick})$, is relatively high; I could estimate

this as 2% of the time. I talk about Juan Carlos, the King of Spain, much less often. I might estimate this, $P(X_{\text{Juan Carlos}})$, as .002%. Luckily for our estimation, P(Y) can legally be ignored if we are only calculating the most likely X to generate Y, because Y would remain constant across options. We can summarize our knowledge below:

(16) $P(X_{\text{Nick}} \mid \text{'Juan Carlos'}) = P(\text{'Juan Carlos'} \mid X_{\text{Nick}}) \cdot P(X_{\text{Nick}})$
$= .000002 \cdot .02$
$= .00000004$

(17) $P(X_{\text{Juan Carlos}} \mid \text{'Juan Carlos'}) = P(\text{'Juan Carlos} \mid X_{\text{Juan Carlos}}) \cdot P(X_{\text{Juan Carlos}})$
$= .2 \cdot .00002$
$= .000004$

Since, $P(X_{\text{Juan Carlos}} \mid \text{'Juan Carlos'}) > P(X_{\text{Nick}} \mid \text{'Juan Carlos'})$, we can infer reference that the likelier referent of 'Juan Carlos' is the individual Juan Carlos.

We can integrate the use of attention into our Bayesian network in order to infer from an incomplete description. Consider that a referent A is held in attention, although another referent B exists that can be referred to by the same word. If, however, I had seen A and B each only once in my life, then the prior probability of each would be similarly low, as would the conditional probability of the sign, given each referent, i.e. $P(\textit{sign} \mid A) \approx P(\textit{sign} \mid B)$. As some help to this calculation, any prior probability can be equally considered as a conditional probability on some context, i.e. $P(A) = P(A \mid \textit{context})$. That I am attending to A at this moment changes the context in such a way that P(A) is magnified. If we assume that attention serves as magnifying effect on the response to a stimulus, then it might be appropriate to include this effect as a function on the prior probability. Some attention function, $\mathcal{A}$, updates the prior probability based on input from previous responses:

(18) $P(Y)_{t+1} = \mathcal{A}\ [P(Y)_t]$

In want of empirical description, I can only offer an abstraction of how this attention function would work. We can consider that all responses to a certain feature vector *X* could be modeled as a response vector, F(*X*). The change in responses to a feature vector that itself does not change would model one's attention. Assuming that one's attention on every feature has some minimal value above zero, I can offer a general attention function as:

(19) $\mathcal{A} = \dfrac{F(X)_t}{F(X)_{t-1}}$

I demonstrate this procedure through an example of incomplete description. Perhaps I am familiar with only three Portuguese entomologists: João, Teresa, and Paula. We might take the prior probability of each to be the same, say, 1% and the conditional probability of the form 'the Portuguese entomologist', given each referent, to be 33.3%. In this situation, we do not have enough information to choose the referent based on the form:

(20) P(João | 'Portuguese entomologist')
= P('Portuguese entomologist' | João) · P(João)
= .333 · .01
= .00333
P(Teresa | 'Portuguese entomologist') = .00333
P(Paula | 'Portuguese entomologist') = .00333

If, however, we are walking past Teresa's house, our response to the referent Teresa will have increased by the time we hear someone cheer 'the Portuguese entomologist' out the window. If this increase in response is ten-fold, then we could posit A, .1/.01= 10. The most probable referent is now inferred as Teresa, due to an increase in prior probability.

(21) $P(\text{Teresa})_{t+1} = A\ [P(\text{Teresa})_t]$
$= 10[.01] = .1$

P(Teresa | 'Portuguese entomologist')
= P('Portuguese entomologist' | Teresa) · P(Teresa)
= .333 · .1
= .0333
P(Teresa | 'Portuguese entomologist') > P(João | 'Portuguese entomologist'), etc.

In the above manner, incomplete descriptions can be understood as signs that probabilistically indicate a certain referent, given the magnifying parameter of attention within a context. This method benefits from modeling itself after a plausible representation of attention, as an increase in response. It also works more smoothly than does the use of an implicit set within predicate logic, as the latter claim does not naturally demonstrate that attention works as a discrete set.

Despite the allure of probabilistic reasoning to model language use, there are concerns that have yet to leave us. Firstly, although Bayesian inference works well in Artificial Intelligence, it has yet to be demonstrated what the biological correlates of such computation would be. Secondly, in the abstract, it is

straightforward to see how attention pushes an optimal candidate to the fore, but somehow empirical evidence has to support the claimed mechanism of attention. It is not clear how such evidence can be collected. Lastly, there are concerns regarding the practicality of Bayesian inference on a large scale. It seems unlikely that the brain would need to compute all possible referents in order to reach an optimal one. Some soft threshold may be involved that suppresses calculation below a certain attention level. These concerns, among many others, await research on the exchange between communication and attention.

References

Bayes, Thomas. 1763. An essay towards solving a problem in the doctrine of chances. *Philosophical Transactions of the Royal Society of London* 53:370-418.

Donnellan, Keith. 1966. Reference and definite descriptions. *Philosophical Review* 77:281-304.

Frege, Gottlob. 1884. *The Foundations of Arithmetic: A logico-mathematical enquiry into the concept of number*. Oxford: Blackwell.

Frege, Gottlob. 1947. On Sense and Nominatum. In *The Philosophy of Language*, ed. A. P. Martinich, *The Philosophy of Language*, 199-211. Oxford: Oxford University Press.

Knill, David & Alexandre Pouget. 2004. The Bayesian brain: the role of uncertainty in neural coding and computation. *Trends in Neuroscience* 27:712-19

Moore, Tirin & Armstrong, Katherine. 2003. Selective Gating of Visual Signals by Microstimulation of Frontal Cortex. *Nature* 421: 370-373.

Peacocke, Christopher. 1975. Proper names, reference, and rigid designation. In *Meaning, Reference, and Designation*, ed. Simon Blackburn, 109-32. Cambridge: Cambridge University Press.

Russell, Bertrand. 1905. On denoting. *Mind* 14: 479-93.

Russell, Bertrand. 1919. Descriptions. In *The Philosophy of Language*, ed. A. P. Martinich, *The Philosophy of Language*, 221-227. Oxford: Oxford University Press.

Russell, Bertrand. 1957. Mr. Strawson on Referring. In *The Philosophy of Language*, ed. A. P. Martinich, *The Philosophy of Language*, 243-246. Oxford: Oxford University Press.

Russell, Stuart & Norvig, Peter. 2003. *Artifical Intelligence: A Modern Approach*, Upper Saddle River, NJ: Pearson.

Strawson, Peter. 1950. On referring. *Mind* 59: 320-44.